NUMBER SYSTEM

The number system or the numeral system is the system of naming or representing numbers. We know that a number is a mathematical value that helps to count or measure objects and it helps in performing various mathematical calculations. There are different types of number systems in Maths like decimal number system, binary number system, octal number system, and hexadecimal number system.

What is Number System in Maths?

A number system is defined as a system of writing to express numbers. It is the mathematical notation for representing numbers of a given set by using digits or other symbols in a consistent manner. It provides a unique representation of every number and represents the arithmetic and algebraic structure of the figures. It also allows us to operate arithmetic operations like addition, subtraction, multiplication and division.

The value of any digit in a number can be determined by:

- The digit
- Its position in the number
- The base of the number system

Before discussing the different types of number system examples, first, let us discuss what is a number?

What is a Number?

A number is a mathematical value used for counting or measuring or labelling objects. Numbers are used to perform arithmetic calculations. Examples of numbers are natural numbers, whole numbers, rational and irrational numbers, etc. 0 is also a number that represents a null value.

A number has many other variations such as even and odd numbers, prime and composite numbers. Even and odd terms are used when a number is divisible by 2 or not, whereas prime and composite differentiate between the numbers that have only two factors and more than two factors, respectively.

In a number system, these numbers are used as digits. 0 and 1 are the most common digits in the number system, that are used to represent binary numbers. On the other hand, 0 to 9 digits are also used for other number systems. Let us learn here the types of number systems.

Types of Number Systems

There are various types of number systems in mathematics. The four most common number system types are:

1. Decimal number system (Base- 10)
2. Binary number system (Base- 2)
3. Octal number system (Base-8)
4. Hexadecimal number system (Base- 16)

Now, let us discuss the different types of number systems with examples.

Decimal Number System (Base 10 Number System)

The decimal number system has a base of 10 because it uses ten digits from 0 to 9. In the decimal number system, the positions successive to the left of the decimal point represent units, tens, hundreds, thousands and so on. This system is expressed in decimal numbers. Every position shows a particular power of the base (10).

Example of Decimal Number System:

The decimal number 1457 consists of the digit 7 in the units position, 5 in the tens place, 4 in the hundreds position, and 1 in the thousands place whose value can be written as:

$(1\times10^3) + (4\times10^2) + (5\times10^1) + (7\times10^0)$

(1×1000) + (4×100) + (5×10) + (7×1)

1000 + 400 + 50 + 7

1457

Binary Number System (Base 2 Number System)

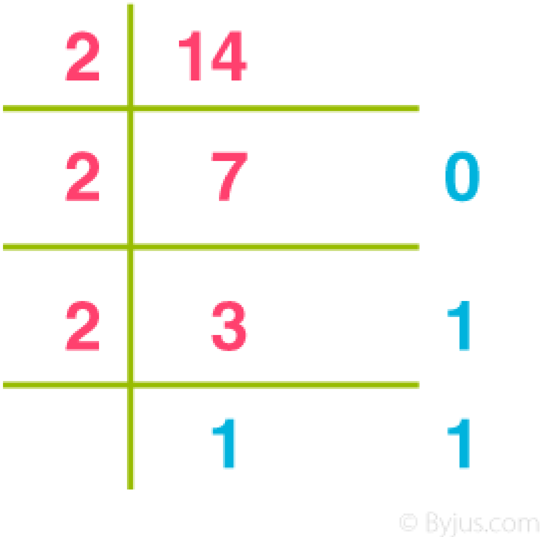
The base 2 number system is also known as the Binary number system wherein, only two binary digits exist, i.e., 0 and 1. Specifically, the usual base-2 is a radix of 2. The figures described under this system are known as binary numbers which are the combination of 0 and 1. For example, 110101 is a binary number.

We can convert any system into binary and vice versa.

Example

Write $(14)_{10}$ as a binary number.

Solution:



Base 2 Number System Example

∴ $(14)_{10} = 1110_2$

Octal Number System (Base 8 Number System)

In the octal number system, the base is 8 and it uses numbers from 0 to 7 to represent numbers. Octal numbers are commonly used in computer applications. Converting an octal number to decimal is the same as decimal conversion and is explained below using an example.

Example: Convert 215_8 into decimal.

Solution:

$215_8 = 2 \times 8^2 + 1 \times 8^1 + 5 \times 8^0$

$= 2 \times 64 + 1 \times 8 + 5 \times 1$

$= 128 + 8 + 5$

$= 141_{10}$

Hexadecimal Number System (Base 16 Number System)

In the hexadecimal system, numbers are written or represented with base 16. In the hexadecimal system, the numbers are first represented just like in the decimal system, i.e. from 0 to 9. Then, the numbers are represented using the alphabet from A to F. The below-given table shows the representation of numbers in the hexadecimal number system.

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Number System Chart

In the number system chart, the base values and the digits of different number systems can be found. Below is the chart of the numeral system.

Number System	Base value	Set of digits	Example
Base 3	3	0, 1, 2	$(123)_3$
Base 4	4	0, 1, 2, 3	$(145)_4$
Base 5	5	0, 1, 2, 3, 4	$(425)_5$
Base 6	6	0, 1, 2, 3, 4, 5	$(225)_6$
Base 7	7	0, 1, 2, 3, 4, 5, 6	$(1205)_7$
Base 8	8	0, 1, 2, 3, 4, 5, 6, 7	$(105)_8$
Base 9	9	0, 1, 2, 3, 4, 5, 6, 7, 8	$(25)_9$
Base 10	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	$(1125)_{10}$

Number System Chart

Number System Conversion

Numbers can be represented in any of the number system categories like binary, decimal, hexadecimal, etc. Also, any number which is represented in any of the number system types can be easily converted to another.

The general representation of number systems are;

Decimal Number – Base 10 – N_{10}
Binary Number – Base 2 – N_2
Octal Number – Base 8 – N_8
Hexadecimal Number – Base 16 – N_{16}

Number System Conversion Table

Binary Numbers	Octal Numbers	Decimal Numbers	Hexadecimal Numbers
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	10	A
1011	13	11	B
1100	14	12	C
1101	15	13	D
1110	16	14	E
1111	17	15	F

Number System Conversion Methods

Number system conversions deal with the operations to change the base of the numbers. For example, to change a decimal number with base 10 to binary number with base 2. We can also perform the arithmetic operations like addition, subtraction, multiplication on the number system. Here, we will learn the methods to convert the number of one base to the number of another base starting with the decimal number system. The representation of number system base conversion in general form for any base number is;

$(Number)_b = d_{n-1}\, d_{n-2}$ —— $.d_1\; d_0\,.\, d_{-1}\, d_{-2}$ —- d_{-m}

In the above expression, $d_{n-1}\, d_{n-2}$ —— $.d_1\; d_0$ represents the value of integer part and $d_{-1}\, d_{-2}$ —- d_{-m} represents the fractional part.

Also, d_{n-1} is the Most significant bit (MSB) and d_{-m} is the Least significant bit (LSB).

Now let us learn, conversion from one base to another.

Decimal to Other Bases

Converting a decimal number to other base numbers is easy. We have to divide the decimal number by the converted value of the new base.

Decimal to Binary Number:

Suppose if we have to convert decimal to binary, then divide the decimal number by 2.

Example 1. Convert $(25)_{10}$ to binary number.

Solution: Let us create a table based on this question.

Operation	Output	Remainder
25 ÷ 2	12	1(MSB)
12 ÷ 2`	6	0
6 ÷ 2	3	0
3 ÷ 2	1	1
1 ÷ 2	0	1(LSB)

Therefore, from the above table, we can write,

$(25)_{10} = (11001)_2$

Decimal to Octal Number:

To convert decimal to octal number we have to divide the given original number by 8 such that base 10 changes to base 8. Let us understand with the help of an example.

Example 2: Convert 128_{10} to octal number.

Solution: Let us represent the conversion in tabular form.

Operation	Output	Remainder
128÷8	16	0(MSB)
16÷8	2	0
2÷8	0	2(LSB)

Therefore, the equivalent octal number = 200_8

Decimal to Hexadecimal:

Again in decimal to hex conversion, we have to divide the given decimal number by 16.

Example 3: Convert 128_{10} to hex.

Solution: As per the method, we can create a table;

Operation	Output	Remainder
128÷16	8	0(MSB)
8÷16	0	8(LSB)

Therefore, the equivalent hexadecimal number is 80_{16}

Here MSB stands for a Most significant bit and LSB stands for a least significant bit.

Other Base System to Decimal Conversion

Binary to Decimal:

In this conversion, binary number to a decimal number, we use multiplication method, in such a way that, if a number with base n has to be converted into a number with base 10, then each digit of the given number is multiplied from MSB to LSB with reducing the power of the base. Let us understand this conversion with the help of an example.

Example 1. Convert $(1101)_2$ into a decimal number.

Solution: Given a binary number $(1101)_2$.

Now, multiplying each digit from MSB to LSB with reducing the power of the base number 2.

$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 0 + 1$

$= 13$

Therefore, $(1101)_2 = (13)_{10}$

Octal to Decimal:

To convert octal to decimal, we multiply the digits of octal number with decreasing power of the base number 8, starting from MSB to LSB and then add them all together.

Example 2: Convert 22_8 to decimal number.

Solution: Given, 22_8

$2 \times 8^1 + 2 \times 8^0$

$= 16 + 2$

$= 18$

Therefore, $22_8 = 18_{10}$

Hexadecimal to Decimal:

Example 3: Convert 121_{16} to decimal number.

Solution: $1 \times 16^2 + 2 \times 16^1 + 1 \times 16^0$

= 16 x 16 + 2 x 16 + 1 x 1

= 289

Therefore, $121_{16} = 289_{10}$

Hexadecimal to Binary Shortcut Method

To convert hexadecimal numbers to binary and vice versa is easy, you just have to memorize the table given below.

Hexadecimal Number	Binary	Hexadecimal Number	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

You can easily solve the problems based on hexadecimal and binary conversions with the help of this table. Let us take an example.

Example: Convert $(89)_{16}$ into a binary number.

Solution: From the table, we can get the binary value of 8 and 9, hexadecimal base numbers.

8 = 1000 and 9 = 1001

Therefore, $(89)_{16} = (10001001)_2$

Octal to Binary Shortcut Method

To convert octal to binary number, we can simply use the table. Just like having a table for hexadecimal and its equivalent binary, in the same way, we have a table for octal and its equivalent binary number.

Octal Number	Binary
0	000
1	001
2	010
3	011

Octal Number	Binary
4	100
5	101
6	110
7	111

Example: Convert $(214)_8$ into a binary number.

Solution: From the table, we know,

2 → 010

1 → 001

4 → 100

Therefore, $(214)_8 = (010001100)_2$

With the help of the different conversion procedures explained above, now let us discuss in brief about the conversion of one number system to the other number system by taking a random number.

Assume the number 349. Thus, the number 349 in different number systems is as follows:

The number 349 in the binary number system is 101011101

The number 349 in the decimal number system is 349.

The number 349 in the octal number system is 535.

The number 349 in the hexadecimal number system is 15D

Number System Solved Examples

Example 1:

Convert $(1056)_{16}$ to an octal number.

Solution:

Given, 1056_{16} is a hex number.

First we need to convert the given hexadecimal number into decimal number

$(1056)_{16}$

$= 1 \times 16^3 + 0 \times 16^2 + 5 \times 16^1 + 6 \times 160$
$= 4096 + 0 + 80 + 6$
$= (4182)_{10}$
Now we will convert this decimal number to the required octal number by repetitively dividing by 8.

8	4182	Remainder
8	522	6
8	65	2
8	8	1
8	1	0
	0	1

Therefore, taking the value of the remainder from bottom to top, we get;

$(4182)_{10} = (10126)_8$

Therefore,

$(1056)_{16} = (10126)_8$

Example 2:

Convert $(1001001100)_2$ to a decimal number.

Solution:

$(1001001100)_2$

$= 1 \times 2^9 + 0 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$

$= 512 + 64 + 8 + 4$

$= (588)_{10}$

Example 3:

Convert 10101_2 into an octal number.

Solution:

Given,

10101_2 is the binary number

We can write the given binary number as,

010 101

Now as we know, in the octal number system,

010 → 2

101 → 5

Therefore, the required octal number is $(25)_8$

Example 4:

Convert hexadecimal 2C to decimal number.

Solution:

We need to convert $2C_{16}$ into binary numbers first.

2C → 00101100

Now convert 00101100_2 into a decimal number.

$101100 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$

$= 32 + 8 + 4$

$= 44$